FTEJerez
Flight Training Europe

Pre-Test
Guidance Material

Version 2.0

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2 Pre Test Guidance Material

2.1 Introduction

Knowledge of elementary mathematics and physics is an essential for success in both the ground examinations and when working as a commercial pilot.

The assessment tests conducted at **FTEJerez** are designed to measure your ability to do the course and to establish your base level of knowledge. This study material is designed simply to act as a **guide to the sort of things which may be included** in the **FTEJerez** assessment and does not necessarily cover all areas which may be tested; **you are encouraged to do further study before assessment**.

A number of self-test questions are included to give you an idea of your understanding of the material.

3 Contents

3.1 Simple Arithmetic

The rules of simple arithmetic are too well-known to need any further repetition here. However, the use of both multiple arithmetic processes and indices can lead to some confusion. This section contains a brief reminder of those procedures.

3.2 Multiple Arithmetic Processes

If a calculation requires the use of a number of the basic arithmetic processes, the rules for the order in which those processes must be completed is as follows:

- Take out any brackets (the rules that follow must be obeyed for any expressions within the brackets).
- Complete any multiplications and divisions from left to right.
- Complete any additions and subtractions from left to right.

For example,

$$(6 \times 3 - 2) \div 4 + 7 \times 8 - 3 = (18 - 2) \div 4 + 7 \times 8 - 3 = 16 \div 4 + 7 \times 8 - 3 = 4 + 56 - 3 = 60 - 3 = \mathbf{57}$$

It is also worth remembering that, when using fractions, division by a fraction is the same process as multiplication by the reciprocal of that fraction.

For example,

$$\frac{1}{7} \div \frac{3}{7} = \frac{1}{7} \times \frac{7}{3} = \frac{(1 \times 7)}{(7 \times 3)} = \frac{7}{21} = \mathbf{\frac{1}{3}}$$

To convert a fraction to a decimal, simply divide the top (numerator) by the bottom (denominator).

Hence,

$$\frac{6}{7} = 6 \div 7 = \mathbf{0.8571}$$

3.3 Powers and Roots

If a quantity is multiplied by itself, it is said to be raised to the second power or squared. If the quantity is multiplied by itself three times it is said to be cubed and so on.

For example,

$$2 \times 2 = 2^2 \quad \text{(two squared)}$$
$$2 \times 2 \times 2 = 2^3 \quad \text{(two cubed)}$$

The figure above and to the right of the quantity in question is called the **index** and it indicates the **power** to which the quantity must be raised. The quantity itself is referred to as the **base**.

Therefore, $2^5 = 32$ and 32 is said to be the **fifth power of 2** and $2^2 = 4$ means that 4 is the **square of 2**. By the same token, 2 is said to be the **fifth root of 32** or the **square root of 4** and these relationships would be written mathematically as follows:

$$2 = \sqrt[5]{32} \quad \text{or} \quad 32^{\frac{1}{5}} \quad \text{and} \quad 2 = \sqrt{4} \quad \text{or} \quad 4^{\frac{1}{2}}$$

The index may also be negative. For example,

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$$

3.4 Index Laws

When powers of the same base are multiplied together, the result can be found by simply adding the powers together.

Thus, $2^3 \times 2^4 = 2^{3+4} = 2^7 = 128$

If powers of the same base are divided, then the powers are subtracted to give the result.

Thus, $\frac{2^5}{2^7} = 2^{5-7} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$

3.5 Algebra

The use of formulae in the calculation of values means that a little basic algebra is needed in navigation, principles of flight and other allied subjects. In simple equations it is usually necessary to move some known values from one side to the other so that the unknown one is left on its own. This process is called **transposition**.

The rules of transposition are quite straightforward. The left-hand and right-hand sides of the equation may be interchanged without affecting the validity of the equation.

Thus, if $a + b = c$ then $c = a + b$.

Whenever a quantity is moved from one side of an equation to the other, the sign of that quantity must be changed. Hence, a plus becomes a minus, a multiply a divide and so on.

Thus, if $a + b = c$ then $a = c - b$ and if $a \times b = c$ then $a = \frac{c}{b}$.

Whatever mathematical operation is carried out on one side of the equation must be carried out on the other side for the equation to remain valid.

Thus, if $a = b + c$ then $a^2 = (b + c)^2$, $a - d = b + c - d$, $3 \times a = 3 \times (b + c)$ and $\frac{1}{a} = \frac{1}{(b+c)}$.

A general example putting these rules together is as follows:

Given the formula: $z = d\left(\frac{a+b-y}{c}\right)^{\frac{1}{2}}$ make a the subject of the formula.

The steps in the transposition of the formula are as follows:

1 Square each side of the formula:

$$z^2 = \frac{d^2(a+b-y)}{c}$$

2 Multiply both sides by c:

$$z^2 \times c = d^2(a+b-y)$$

3 Divide both sides by d^2:

$$\frac{z^2 \times c}{d^2} = \frac{d^2(a+b-y)}{d^2}$$

and from that we may say:

$$\frac{z^2 \times c}{d^2} = a+b-y$$

4 Move $(b-y)$ from the *right-hand side* to the *left-hand side* (remember to change the signs of the variables):

$$\frac{z^2 \times c}{d^2} - b + y = a$$

5 Finally, simply rewrite the formula with a on the left-hand side:

$$a = \frac{z^2 \times c}{d^2} - b + y$$

The value of a could now be found given the values of the other variables.

Some common aeronautical examples of the transposition of formulae are as follows:

In aerodynamics, lift and drag are dependent on density (ρ), true airspeed (V), wing area (S) and variables known as the coefficients of lift (C_L) and drag (C_D). Lift can now be expressed by the formula:

$$L = C_L \, \frac{1}{2}\rho \, V^2 \, S$$

Hence, transposing the formula to express V in terms of the other variables gives:

$$V = \left(\frac{L}{C_L \frac{1}{2}\rho \, S}\right)^{\frac{1}{2}}$$

Recall: $\sqrt{a} = a^{\frac{1}{2}}$

V could now be calculated if the values of the variables on the right-hand side of the formula were known.

In another problem it is sometimes necessary to calculate the maximum theoretical range of a radio facility. The formula used is:

$$MTR = 1.25(\sqrt{H_{Tx}} + \sqrt{H_{Rx}})$$

Where, MTR is the maximum theoretical range, H_{Tx} is the height of the transmitter above mean sea level and H_{Rx} is the height of the receiver (usually the aircraft).

The formula can be transposed to make H_{Rx} the subject if the required height of the aircraft needs to be found in order to receive a transmission at a given range. Thus:

$$H_{Rx} = \left(\frac{MTR}{1.25} - \sqrt{H_{Tx}}\right)^2$$

H_{Rx} could now be found if the values of the variables on the right-hand side of the formula were known.

3.6 Ratios

When two quantities are compared, the result of dividing one by the other is called the ratio of the first quantity to the second. For example:

The ratio of 6 to $3 = \frac{6}{3}$ or $\frac{2}{1}$

This can be expressed in words as:

The ratio of 6 to 3 is the same as the ratio of 2 to 1

Or symbolically:

$$6 : 3 \equiv 2 : 1$$

When the ratio between two quantities is known it may be used to calculate the particular value of one quantity corresponding to a given value of the other quantity.

For example:

The specific gravity, SG, of a fuel is the ratio of the mass of a given volume of the fuel to the mass of the same volume of water. The SG depends upon the chemical composition of the fuel, but a commonly used value for aviation fuel is 0.73. We know that one imperial gallon of water has a mass of $10\ lbs$; therefore, if we have $500\ imperial\ gallons$ of this fuel we can calculate the mass of the fuel as follows:

1. $500\ imperial\ gallons$ of water has a mass of $5000\ lbs$,
2. Then if the mass of the $500\ imperial\ gallons$ of the fuel is $m\ lbs$,

$$\frac{0.73}{1} = \frac{m}{5000}$$

3. Therefore,

$$m = 0.73 \times 5000 = 3650\ lbs$$

3.7 Percentages

When the ratio of one quantity to another is expressed with the denominator 100, the corresponding numerator gives the percentage that the first quantity is of the second.

For example:

$$6:3 \equiv 2:1 \equiv 200:100 \text{ or, 6 is 200 per cent of 3 or 200\% of 3}$$

To convert a ratio to a percentage, simply convert the ratio to a decimal and multiply the result by 100. Thus:

$$\frac{3}{8} = 0.375$$

$$0.375 \times 100 = 37.5$$

Therefore,

$$3 \; is \; 37.5\% \; of \; 8$$

To find the required percentage of a given quantity simply divide the percentage by 100 and multiply the result by the given quantity.

For example, find 60% of 22.

$$\frac{60}{100} = 0.6$$

Therefore,

$$60\% \; of \; 22 = 0.6 \times 22 = 13.2$$

When solving problems involving percentages it is important to know of which quantity the percentage is to be found. The following two examples illustrate the errors that can arise unless clear thinking is used. The first example is straightforward, the second example is not!!

For example: The required level runway length for an aircraft to take-off safely under specified meteorological and aircraft configuration conditions is $4000\,ft$. However, the runway in use actually has an upslope of 1° and this increases the required runway length by 10%. Calculate the runway length required under these circumstances.

$$Required\ runway\ length = 4000 + 10\%\ \ of\ 4000 = 4000 + (4000 \times 0.1) = 4000 + 400 = \mathbf{4400\ ft}$$

One more example: The total fuel capacity of an aircraft is $2000\ gallons$. If 25% of the fuel used in a flight must be kept in reserve for emergencies, calculate the maximum amount of fuel available for a flight, assuming that the reserve fuel is unused.

The most common error is to say that the reserve fuel is 25% of the total fuel ($2000\ gallons$). This gives the reserve fuel as $0.25 \times 2000 = 500\ gallons$, leaving $2000 - 500 = 1500\ gallons$ as the maximum flight fuel available. This is incorrect!

The correct starting point for the calculation is that:

$$Total\ fuel = Flight\ fuel + Reserve\ fuel = Flight\ fuel + 25\%\ of\ Flight\ fuel$$

$$2000 = Flight\ fuel + (0.25 \times Flight\ fuel) = 1.25 \times Flight\ fuel$$

$$\frac{2000}{1.25} = Flight\ fuel$$

$$Maximum\ available\ flight\ fuel = 1600\ gallons$$

3.8 Variation

If we take an expression such as $a = \frac{b}{c}$ then it is possible to state how one variable in the expression will vary with a change in the value of one of the other variables in the expression.

So, in the expression above we can say that if b increases then a will also increase, and if b decreases then a will also decrease (assuming c to be constant).

From the above deductions we say that ***a* varies directly as *b***.

On the other hand if c increases then a will decrease, and if c decreases then a will increase (assuming b to be constant).

From these deductions we say that ***a* varies inversely or indirectly as *c***.

If two quantities are known to vary directly or inversely it is possible to calculate the change in one quantity corresponding to a known change of the other quantity, if the original values of the two quantities are known. This is of great value when a complicated formula has been solved for a given set of quantities and the new result is required when there is a change of only one or two of the original quantities.

The rules that must be remembered are:

When two quantities vary **directly** the result of dividing one into the other is a constant. If a varies directly as b then:

$$\frac{Old\ a}{Old\ b} = \frac{New\ a}{New\ b} = constant$$

This is most usefully expressed in the format:

$$New\ a = Old\ a \times \frac{New\ b}{Old\ b}$$

When two quantities vary **inversely** the result of multiplying them together is a constant. If a varies inversely as c then:

$$Old\ a \times Old\ c = New\ a \times New\ c = constant$$

This is most usefully expressed in the format:

$$New\ a = Old\ a \times \frac{Old\ c}{New\ c}$$

For example:

Given the expression $P = \frac{27.3 \times R \times T}{Q \times M}$ and that P is 12 when R is 7. Calculate the value of P when R decreases to 5, if the other terms remain constant. From the expression, P varies directly as R.

Therefore, $New\ P = Old\ P \times \frac{New\ R}{Old\ R} = 12 \times \frac{5}{7} = \frac{60}{7} = 8.5714$ (P and R both decrease).

Using the same expression and given that M is 2 when P is 15, calculate the value of M when P increases to 20, if the other terms remain constant. From the expression, P varies inversely as M.

Therefore, $New\ M = Old\ M \times \frac{Old\ P}{New\ P} = 2 \times \frac{15}{20} = 1.5$ (M decreases when P increases).

3.9 Elementary Geometry

3.9.1 The Triangle

The triangle is, by definition, a three-angled and three-sided figure.

- It is a **scalene** triangle if the three sides are of unequal length.
- It is an **isosceles** triangle if two of the sides are of equal length -but the remaining side is not.
- It is an **equilateral** triangle if the three sides are equal in length.
- It is a **right-angled** triangle if one of the angles equals $90°$.

The following properties of a triangle must be remembered:

- The sum of the angles of the triangle is $180°$.
- In an isosceles triangle, the two angles opposite the equal sides are also equal.
- In an equilateral triangle, the three angles are all $60°$.
- It is customary to annotate the triangle in the following manner; the angles are given capital letters and the sides opposite those angles are given the corresponding lower case letters.

3.9.2 The Circle

By definition, the circle is the closed figure obtained by rotation of a constant length about a fixed point that is the centre of the circle. The other parts of the circle with which students should be familiar are:

- The **circumference** is the distance around the perimeter or outside edge of the circle.
- The **radius** is the distance from the centre of the circle to any point on its perimeter.
- The **diameter** is the distance across the circle along a straight line passing through the centre; it is equal to twice the radius.
- An **arc** is any part of the circumference.
- A **semicircle** is an arc that is half the circumference.
- A **chord** is a straight line drawn across the circle that does not pass through the centre.
- A **sector** is a part of the circle contained between two radii and the enclosed arc.
- A **segment** is a part of the circle contained between a chord and the enclosed arc.
- A **tangent** is a straight line that just touches the circumference at a point known as the point of tangency; it does not cut the circumference.

Figure 1 -following page- illustrates the majority of the parts of the circle defined above.

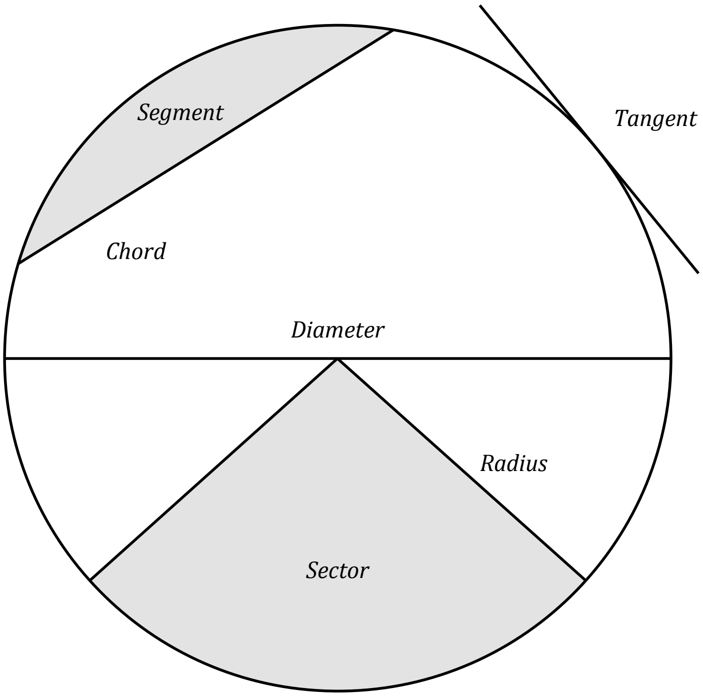


Figure 1. The Circle.

3.9.2.1 Properties of the Circle

- The circumference of the circle (c) is: $c = 2\pi r = \pi D$, where r is the radius and D is the diameter.
- The area (a) of the circle is: $a = \pi r^2$

3.9.3 General Geometric Properties

The following geometrical truth or theorems should be remembered.

- Vertically opposite angles are equal

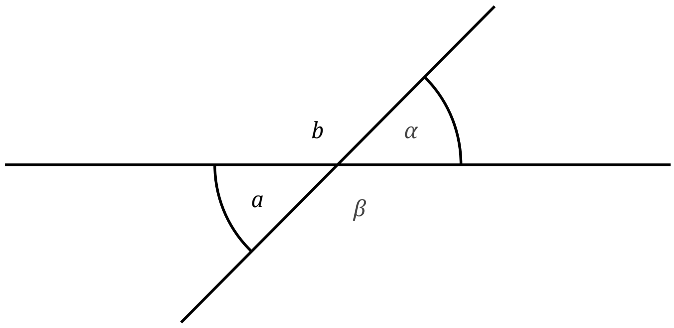


Figure 2. Opposite angles.

Then in the diagram above: $a = \alpha$ and $b = \beta$

- If the horizontal lines are parallel, corresponding angles are equal.

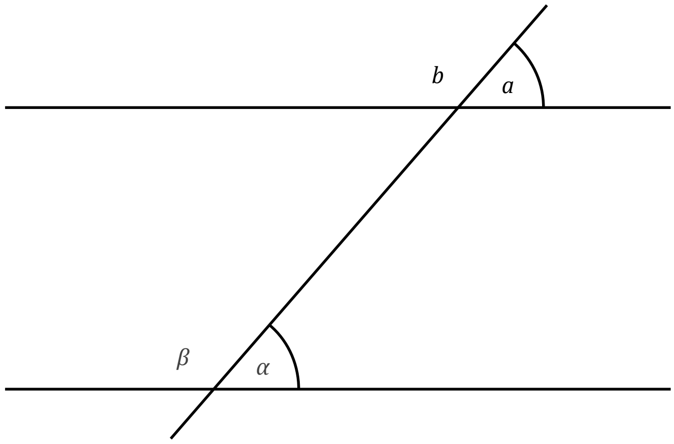


Figure 3.

Then in the diagram above: $a = \alpha$ and $b = \beta$

- If the horizontal lines are parallel, alternate angles are equal

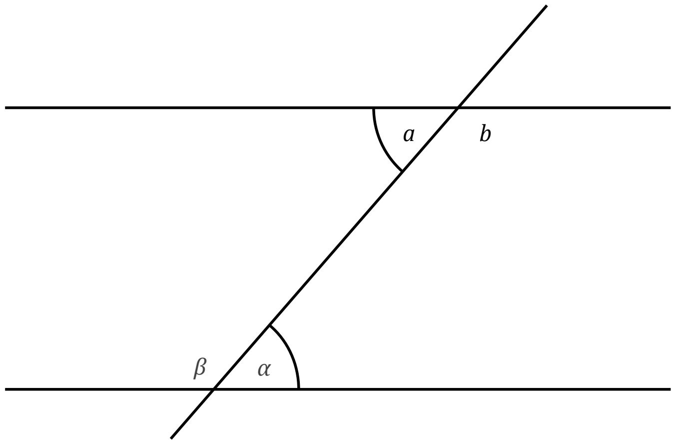


Figure 4.

Then in the diagram above: $a = \alpha$ and $b = \beta$

- The Pythagoras' Theorem states that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides. The hypotenuse is the side opposite the right angle and is the longest side of the triangle (see Figure 5. in the following page).

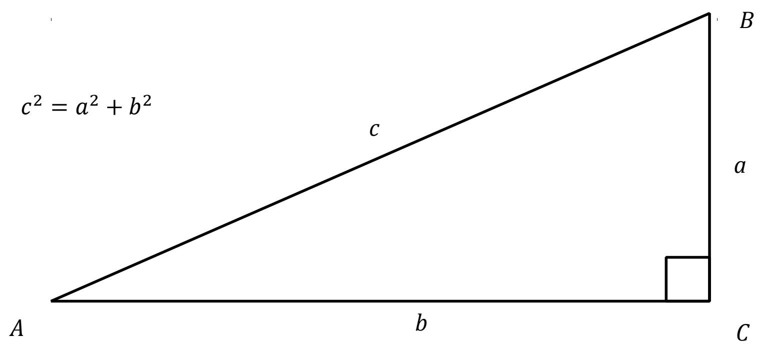


Figure 5. Pythagoras' Theorem.

Where angle $C = 90°$ and c is the hypotenuse.

- Any tangent to a circle is at right angles to the radius at the point of tangency.

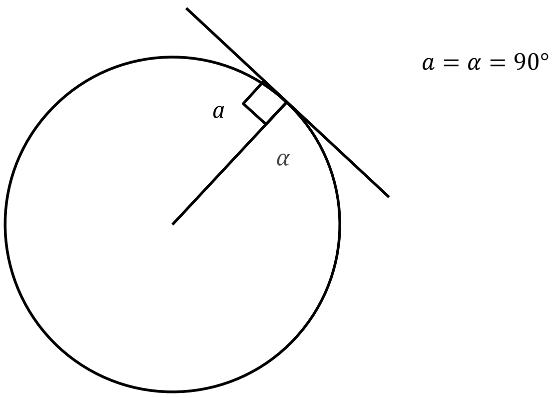


Figure 6. Point of Tangency.

3.10 Circles

The circle is divided into $360°$ equal sectors. The angle between each pair of radii is known as **one degree or 1°**.

Each degree is divided into 60 equal parts known as **minutes** (’) and each minute is further divided into 60 equal parts known as **seconds** (”).

Any angle is then described in those units. For example:

49 degrees, 27 minutes, 12 seconds would be written 49° 27’ 12”

This method of circular measurement is commonly used in mathematics and navigation.

3.11 Trigonometry

A working knowledge of trigonometry is required to succeed in a number of the JAR examinations. This section will cover those elements of basic trigonometry considered essential for the successful completion of the course.

3.11.1 Similar Triangles

When three pairs of corresponding angles enclosed in two triangles are equal, the triangles are said to be **similar**. This does not necessarily imply that the two triangles are of the same size, as shown below:

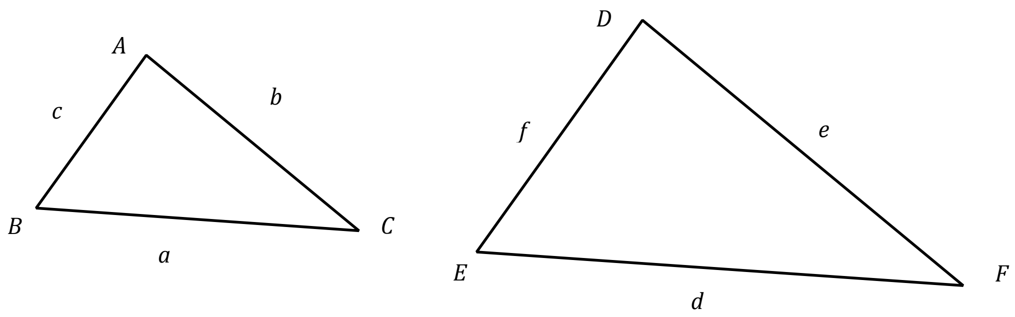


Figure 7. Similar Triangles.

Triangles ABC and DEF are similar because angle A = angle D, angle B = angle E and angle C = angle F, but, $a \neq d$, $b \neq e$ and $c \neq f$. However, the ratios of the corresponding sides are equal.

3.11.2 Ratios of the Sides of Similar Triangles

In the diagrams:

$$a : d \equiv b : e \equiv c : f$$

$$or \quad \frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

$$or \quad \frac{d}{a} = \frac{e}{b} = \frac{f}{c}$$

From these relationships it is also true to say that the ratios of two corresponding sides in each triangle are equal:

$$a : b \equiv d : e$$
$$a : c \equiv d : f$$
$$b : c \equiv e : f$$

$$or \quad \frac{a}{b} = \frac{d}{e}, \quad \frac{a}{c} \equiv \frac{d}{f}, \quad \frac{b}{c} \equiv \frac{e}{f}$$

$$or \quad \frac{b}{a} = \frac{e}{d}, \quad \frac{c}{a} \equiv \frac{f}{d}, \quad \frac{c}{b} \equiv \frac{f}{e}$$

Thus if the sides of a triangle are known and one side of a similar triangle is also known, it is possible to calculate the values of the other two sides.

For example:

If $a = 7,\ b = 5,\ c = 4$ and $d = 11$, calculate e and f.

$$\frac{e}{d} = \frac{b}{a} \quad and \quad \frac{f}{d} = \frac{c}{a}$$

Therefore,

$$e = d \times \frac{b}{a} \quad and \quad f = d \times \frac{c}{a}$$

$$e = 11 \times \frac{5}{7} \quad and \quad f = 11 \times \frac{4}{7}$$

$$e = 7.8571 \quad and \quad f = 6.2857$$

3.12 Trigonometric Functions

The three sides of a right-angled triangle are given names relative to the included angle under consideration. The adjacent side is the side alongside the angle, the opposite side is just that and the hypotenuse has already been defined. The diagram below illustrates the concepts.

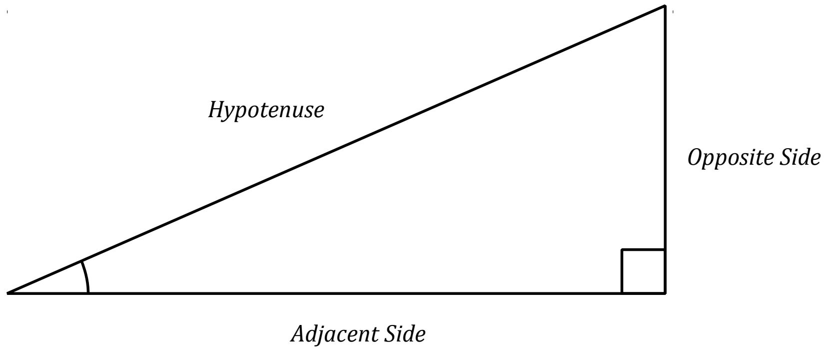


Figure 8. Trigonometric concepts.

The common trigonometric functions are as follows:

$$Sine\ (Sin) = \frac{Opposite\ side}{Hypotenuse}$$

$$Cosine\ (Cos) = \frac{Adjacent\ side}{Hypotenuse}$$

$$Tangent\ (Tan) = \frac{Opposite\ side}{Adjacent\ side}$$

$$Cosecant\ (Cosec) = \frac{1}{Sine} = \frac{Hypotenuse}{Opposite\ side}$$

$$Secant\ (Sec) = \frac{1}{Cosine} = \frac{Hypotenuse}{Adjacent\ side}$$

$$Cotangent\ (Cot) = \frac{1}{Tangent} = \frac{Adjacent\ side}{Opposite\ side}$$

3.13 Scalar and Vector Quantities

A scalar quantity is one that is fully defined by a magnitude only. For example, mass is a scalar as is volume. Two similar scalar quantities can be added by simply adding the individual values together. Thus a mass of 20 kilograms plus a mass of 30 kilograms is simply 50 kilograms.

A vector quantity however, is one that is defined by both a magnitude and a direction. The most common vector quantities in aviation are velocity, acceleration and force. Hence, an aircraft velocity is only defined fully when both magnitude and direction are specified. So an aircraft velocity might be given as 300 knots on a heading of $320°$. Wind speeds are given in the form $310/20$ where $310°$ is the direction the wind is coming from and 20 is its speed in knots.

Vector addition is a somewhat more complicated problem than scalar addition and is very often carried out by the use of a diagram. The most common application in the aviation field is the vector addition of aircraft true airspeed/heading and wind speed to give aircraft groundspeed and track.

A typical problem would be to find the aircraft groundspeed and track given that the aircraft's true airspeed was 75 knots, its heading $130°$ and the wind velocity $225°/25$.

Graphically, the solution could be found using the diagram below **if it were drawn accurately to an appropriate scale**. (Note: winds blow from the given direction whereas aircraft head towards the given direction).

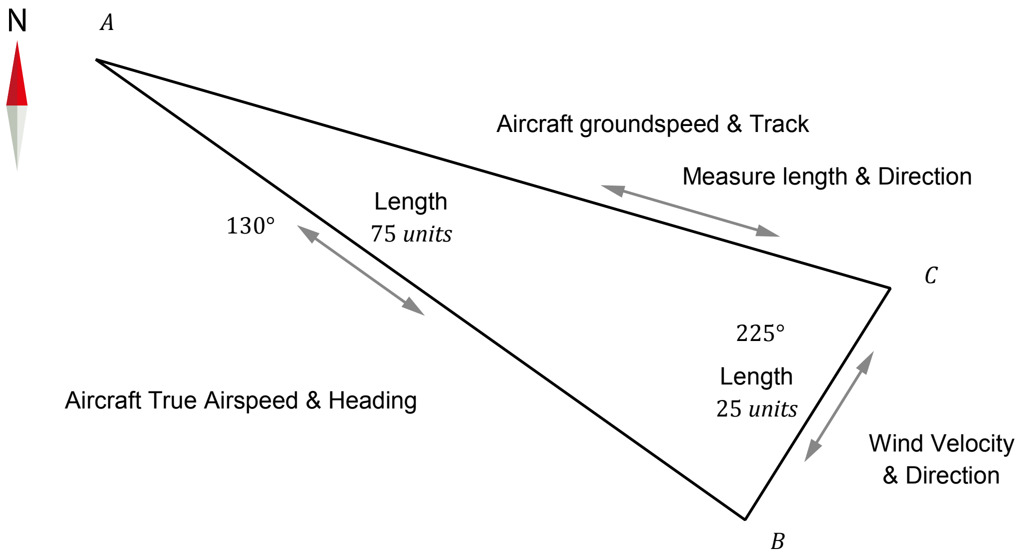


Figure 9.

It is very important to note that vectors can be added diagrammatically only when the arrows that define their directions follow one another around. The magnitude of the vector sum of the vectors is given by the length of the line that completes the triangle and the **direction of the vector sum is in the opposite direction from the sense of the two vectors being added** (called the component vectors).

In essence if the aircraft points on a heading to take it from A to B it will be 'pushed' by the wind and end up at point C.

Therefore, assuming that the same scale is used for both the wind speed and the aircraft true airspeed, and that north is measured directly up the paper, the aircraft groundspeed and track would be approximately (by simple measurement) 80 knots and $110°$. Any number of vectors can be added in this way providing the convention of the directions of those vectors following one another is obeyed. The vector sum is found by joining the beginning of the line representing the first vector component to the end of the line representing the last vector drawn. Once again the direction of the sum reverses the sense of the components.

3.14 Resolution of Vectors

Vectors can also be 'resolved' into component vectors by effectively reversing the process of addition previously explained. Once again it is common to carry out the process diagrammatically and it is normal to resolve a vector into only two components that are usually at right angles to one another.

For example, in Principles of Flight, the movement of air over an aerofoil surface (the aircraft wing) produces pressure changes that create an aerodynamic force (a vector) called the Total Reaction. Lift is defined as the component of the Total Reaction that acts at $90°$ to the relative airflow over the wing and drag as the component of the Total Reaction that acts in the same direction as the relative airflow. The Total Reaction can be resolved into its components (lift and drag) diagrammatically and the principle is illustrated in the diagram below.

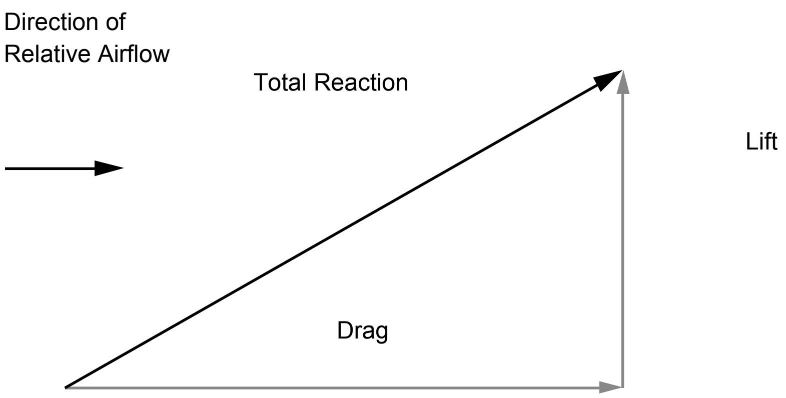


Figure 10.

The Total Reaction has been 'resolved' into its two 'components' Lift and Drag. It can be seen that the sum of Lift and Drag, using the vector addition method previously explained, is the Total Reaction.

3.15 Definitions of Common Physical Properties

The following physical properties are commonly referred to in one or more of the JAR topics:

- The **velocity** of a body is defined as its rate of change of position with respect to time, the direction of motion being specified. If the body is travelling in a straight line it is in linear motion, and if it covers equal distances in equal successive time intervals it is in **uniform linear motion**. For uniform velocity, where s is the distance covered in time t, the velocity is given by:

$$v = \frac{s}{t}$$

  The SI unit of velocity is ms^{-1} (metres per second); in aviation, the unit most commonly used is the $knot$ (one nautical mile per hour).

- The **acceleration** of a body is its rate of change of velocity with respect to time. Any change in either speed or direction of motion involves acceleration: retardation is merely a negative acceleration. When the velocity of a body changes by equal amounts in equal intervals of time it is said to have a uniform acceleration, measured by the change in velocity in unit time. If the initial velocity u of a body in linear motion changes uniformly in time t to velocity b, its acceleration a is given by:

$$a = \frac{(v-u)}{t}$$

  The SI unit of acceleration is ms^{-2} (metres per second squared).

- The **mass** of a body may be defined as the quantity of matter in the body. As previously stated, the SI unit of mass is kg (kilogram).

- The **density** of a substance is the mass per unit volume of that substance. The SI unit of density is kgm^{-3} (kilogram per cubic metre).

- The **momentum** of a body is defined as the product of its mass and its velocity. Momentum is therefore a vector quantity and a change in either speed or direction constitutes a change in momentum. The SI unit of momentum is $kgms^{-1}$ (kilogram metre per second).

- A **force** is that quantity which when acting on a body that is free to move, produces an acceleration in the motion of that body, and is proportional to the rate of change of momentum of the body. Force (F) is expressed in terms of the mass (m) and acceleration (a) of a body by the formula (derived from Newton's second law of motion): $F = ma$. The SI unit of force is the N (Newton), and, in terms of the fundamental SI units, the unit of force is $kgms^{-2}$ (kilogram metres per second squared). $1\ N$ is the force that gives an acceleration of 1 metre per second squared to a mass of 1 kilogram.

- The **weight** of a body is a measure of the gravitational force of attraction that the earth exerts on its mass. The SI unit of weight is the same as that of force, ie the Newton, and the gravitational constant generally applied is the acceleration due to gravity (g) equal to 9.81 ms^{-2}.

- **Work**. A force is said to do work when its point of application moves, and the amount of work done is the product of the force and the distance moved in the direction of the force. As noted, the unit of work is the J (Joule) and, in terms of the fundamental SI units, the unit of work is kgm^2s^{-2} (kilogram metres squared per second squared). $1\ J$ is the work done when a force of 1 Newton moves a mass of 1 kilogram by 1 metre in the direction of the force.

- **Power** is defined as the rate of doing work and is measured in units of work per unit time. The SI unit of power is the W (Watt) and, in terms of the fundamental SI units, the unit of power is kgm^2s^{-3} (kilogram metres squared per second cubed).

- **Pressure** is the force per unit area acting on the surface of a body. The SI unit of pressure is the Pa (Pascal) and, in terms of the fundamental SI units, the unit of pressure is $kgm^{-1}s^{-2}$ (kilogram per metre per second squared).

- **Inertia** of a body is the tendency of that body to maintain its state of rest or uniform motion in a straight line. Newton's First Law states that the application of an outside force is necessary to overcome the inertia of a body either at rest or in a state of uniform motion.

3.16 Moments and Couples

The moment of a force about a point is the tendency of the force to turn the body to which it is applied about that point. The magnitude of the moment is the product of the force and the perpendicular distance from the point to the line of action of the force and the total moment of a number of forces about a point is given by the sum of the individual moments (the direction of the rotational tendency must clearly be taken into account). The principle of moments is illustrated below. AC is a board resting on a pivot B. A force of $75\ N$ acts vertically upwards through A and a force of $50\ N$ through C.

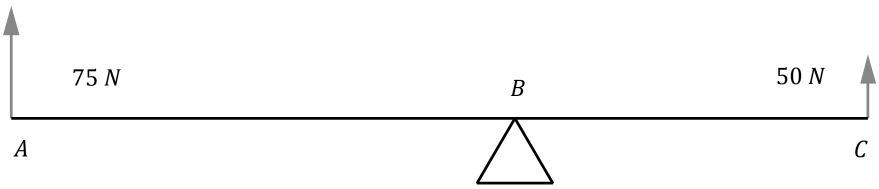


Figure 11.

If $AB = 10\ cm\ (0.10\ M)$ and $BC = 7\ cm\ (0.07\ M)$, taking moments about B will give, if we take the convention that a clockwise rotation is positive and an anticlockwise negative:

$$Total\ moment = (75 \times 0.10) - (50 \times 0.07) = 7.5 - 3.5 = 4\ Nm$$

Because the sign of the total moment is positive, the two forces together would tend to rotate the board AC in a clockwise direction. The magnitude of the moment is a measure of the rate of rotation of the board (the greater the magnitude, the greater the rate of rotation).

If the total moment about a point is zero, the system is said to be in equilibrium with respect to moments. The whole system is in equilibrium if the vector sum of the forces involved is also zero.

A couple is a system of two equal and parallel forces acting in opposite directions but not in the same line. The moment of a couple about any point in the plane of the forces is constant and equal to the product of one of the forces and the distance between the lines of action of the two forces.

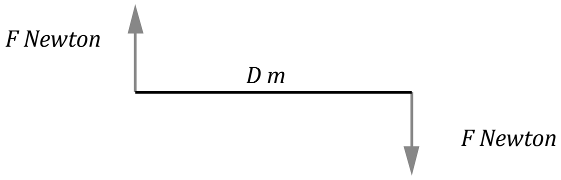


Figure 12.

3.17 Electricity

3.17.1 Ohm's law

Probably the most important mathematical relationship between voltage, current and resistance in electricity is something called Ohm's Law (George Ohm 1827). The Ohm's law formula is used to calculate electrical values so that we can design circuits and use electricity in a useful manner. Ohm's Law is shown below.

$$I = \frac{V}{R}$$

Where I is current, V is voltage and R is resistance. Depending on what you are trying to solve we can rearrange it two other ways.

$$V = I \times R \text{ or } R = \frac{V}{I}$$

3.18 Pressure

Pressure in cylinder $A = \frac{Force\ A}{Area\ A}$

Pressure in cylinder $B = \frac{Force\ B}{Area\ B}$

Pressure in cylinder A = Pressure in cylinder B

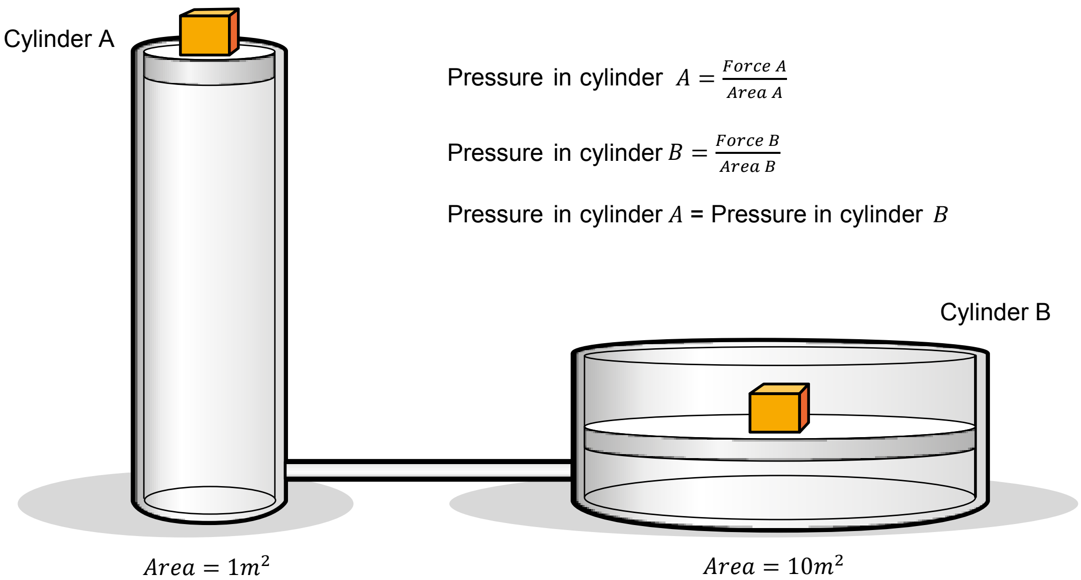


Figure 13.

3.19 Work

$$Work\ Done = Force \times Distance\ Moved$$

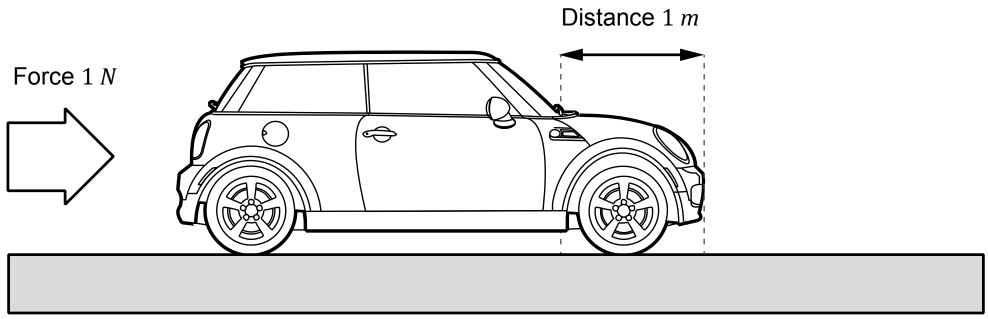
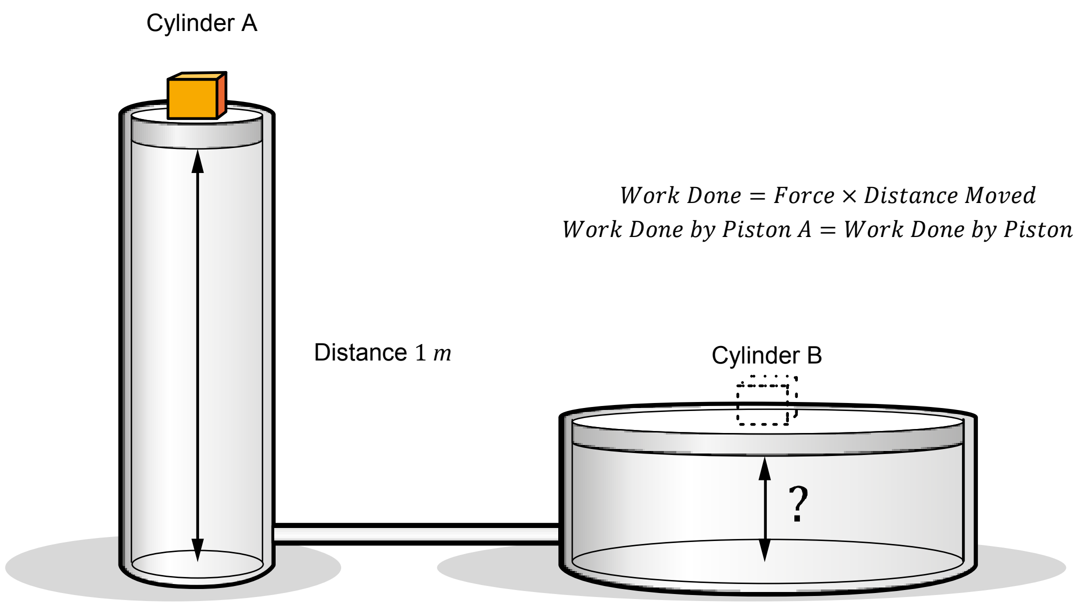


Figure 14.



$$Work\ Done = Force \times Distance\ Moved$$
$$Work\ Done\ by\ Piston\ A = Work\ Done\ by\ Piston\ B$$

Figure 15.

3.20 Light

Light is a form of energy, the speed of light in vacuum is constant at $3 \times 10^8\ m/sec$. Light is subject to Refraction on entering any medium of a different density.

3.21 Review Questions

Find the value of the expressions in questions 1 to 19.

1 $(3 \times 5) - 12 \div (3 + 1)$

2 $18 - (10 \div 2) + 3 \times (5 - 2)$

3 $\left(\left((7 \times 5) - 12\right) + 1\right) \div (4 + 2)$

4 $\frac{11}{12} \div \frac{5}{3}$

5 $\frac{5}{8} + \frac{1}{4} \times \frac{1}{2}$

6 $3 \times \left(\frac{1}{2} + \frac{1}{4}\right)$

7 $3^2 \times 3^{-2}$

8 $\sqrt[3]{5} \times 5^{\frac{2}{3}}$

9 What is 150% of 20?

10 75 is what percentage of 200?

11 75 is an increase of what percentage of 50?

12 The formula used to convert degrees Fahrenheit (°F) to degrees Centigrade (°C) is as follows: $℃ = 5 \times (°F - 32)/9$. Using this formula, find the Fahrenheit equivalent of 50 ℃.

13 In any triangle, using the normal convention for sides and angles:

$$a^2 = b^2 + c^2 - 2 \times b \times c \times cosA$$

Note: it is normal, in mathematical equations, to write $2 \times b \times c \times cosA$, and this convention will be used in the formulae in the rest of these questions.

The formula would now be written as follows:

$$a^2 = b^2 + c^2 - 2\, b\, c\, cosA$$

Using this formula, find the value of A in the triangle below.

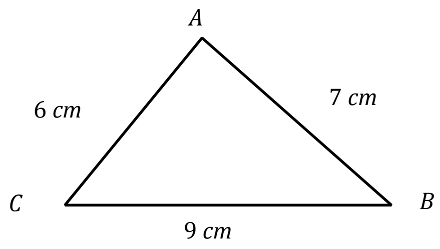


Figure 16.

14 One of the equations of motion resulting from Newton's Laws of Motion is of the form shown below:

$$v^2 = u^2 + 2as$$

Where, v is the final velocity of a body, u is its initial velocity, a is the acceleration and s is the distance travelled during the acceleration period.

Find the distance covered by a body with an initial velocity of $50\ ms^{-1}$, a final velocity of $100\ ms^{-1}$ and an acceleration of $10\ ms^{-2}$.

15 The period of oscillation of a simple pendulum (t seconds) is given by the formula:

$$t = 2\pi(L/g)^{1/2}$$

where, $\pi = 3.142$, L metres is the length of the pendulum and $g\ ms^{-2}$ is the acceleration due to gravity. Find the length of a pendulum whose period is 10 seconds if $g = 9.81\ ms^{-2}$.

16 If the system shown below is in equilibrium, find the value of W.

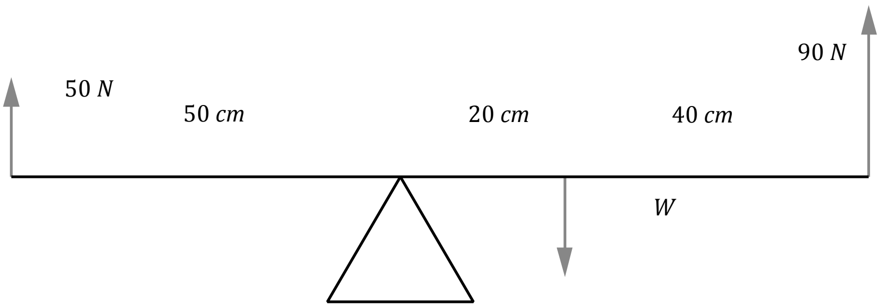


Figure 17.

17 A slope is such that it rises by 1 metre for every 7 metres of horizontal distance covered. By how much will it rise over a horizontal distance of 49 metres?

18 Find the value of the following expression:

$$5^3 - 7^2 + \sqrt{16} \times (45 - 4^2)$$

19 The formula $tan\ \theta = V^2/gr$ relates the radius of turn (r metres) of an aircraft at an angle of bank of $\theta°$ if it is flying at a speed of $V\ ms^{-1}$.

If $1\ knot = 0.514\ ms^{-1}$, find the radius of turn, in metres, of an aircraft at an angle of bank of $45°$ flying at a speed of $300\ knots$.

3.22 Answers

Exercice	Answer
1	12
2	22
3	4
4	0.55 (11/20)
5	0.75 (6/8)
6	2.25 (9/4)
7	1
8	5
9	30
10	37.5%
11	50%
12	122º F
13	87.3º
14	375 m
15	24.85 m
16	-145 N
17	7 m
18	192
19	2423.8 m

3.23 Reasoning

Reasoning is to some extent dependant upon innate (or basic) ability, but it is also a process through which an individual uses other abilities in sequence to solve a problem. A complication in aviation, both in the real world and in the JAA exams, is deciding 'what the question is'. A lot of candidates who achieve poor results or fail the JAA exams do so because they are unable to understand the question.

JAA use a lot of 'indirect questions' and 'negative questions' in order to check understanding and to filter out those who try to get through by remembering lots of detail. In the cockpit a pilot first needs to work out what the information is telling him/her and then work out what the question to be answered is – not everyone is capable of doing this!

3.23.1 Numerical Reasoning

This is a test of your Numerical Reasoning skills, in this type of test the question may seem fairly straightforward or it may not, but the complication is in working out the logical route to be followed.

Read the following paragraphs very carefully together with their associated statements. Use only the information supplied in each paragraph, do not add information from personal knowledge or experience.

1. Air Traffic Control (ATC) instruct a passenger airliner to descend from its cruising altitude of $37{,}000\,ft$ and level off at $10{,}000\,ft$. If the aircraft flies a normal descent at $1500\,ft\ per\ minute$, approximately how long will it take to descend?

- a $28\ min.$
- b $16.5\ min.$
- c $18\ min.$
- d $24.7\ min.$

Here we need to begin by making a logical plan of what we know and how the information links together and then we need to know what we are trying to achieve. From **Known to Unknown**.

1. *Height loss!* The aircraft is at $37{,}000\,ft$ and needs to level at $10{,}000\,ft$ a height loss of $27{,}000\,ft$ $(37{,}000 - 10{,}000)$.
2. *Rate of height loss!* The aircraft will descend $1{,}500\,ft$ every minute, so a height loss of $27{,}000\,ft$ will take $(27{,}000/1{,}500)$ **18 minutes**. So the answer is **c**.

Now try the **second example**.

2. Aircraft A departs Poontap airfield at $1130\ Hr$ and estimates reaching Nangaski $80\ miles$ away at $1210\ Hr$, aircraft B departs Poontap at $1140\ Hr$ and follows aircraft A to Nangaski along the same route. If aircraft B flies $10\ mph$ (miles per hour) faster than aircraft A at approximately what time will aircraft B reach Nangaski?

- a $1118\ Hr$
- b $1238\ Hr$
- c $1228\ Hr$
- d $1217\ Hr$

3.23.2 Verbal Reasoning

Verbal reasoning, sometimes called critical thinking or more simply problem solving, involves similar processes to the numerical reasoning described above but here the answer is far less obvious. In verbal reasoning it is essential to pay attention to the fine detail in the use of words and punctuation; and to ask 'what am I actually being told and what am I not being told?' There is also a huge difference in meaning between words such as could, should, must and can; we need to be precise!

1. Many people like to live in the suburbs rather than the towns and as a result where our ancestors may have walked to work or school we now drive. Motor vehicles have become more readily available and are frequently seen as symbols of success. In the UK it is estimated that the majority of four-wheel drive vehicles are only used for short journeys to school or office and never go off-road. Groups concerned about pollution and Global Warming often cite the 4x4 as major contributors to the 'Greenhouse Gases' which are destroying our atmosphere. They often also claim that other vehicles or pedestrians in collision with a 4x4 are more likely to suffer damage or injury because of the height of the 4x4 and its bumpers.

Consider the following statements and select the most appropriate answer:

i *Four-wheel drive vehicles are responsible for destroying our atmosphere*.

ii *4x4's cause the majority of injuries on the UK roads*.

iii *The majority of four-wheel drive vehicles are only used for short journeys*.

And the answers given are:

a Statements i and ii are incorrect.

b Statements i and ii are correct.

c Statement iii is incorrect.

d Statements i, ii and iii are incorrect.

Here we know quite a lot from the text but how much of it is relevant? Examining the fine detail is essential.

Consider the first statement: *Four-wheel drive vehicles are responsible for destroying our atmosphere*.

What we were actually told was: *Groups… often cite the 4x4 as major contributors to the 'Greenhouse Gases' which are destroying our atmosphere*. Which makes statement i false.

Now, consider the second statement: *4x4's cause the majority of injuries on the UK roads*.

What we were actually told was: *Vehicles or pedestrians in collision with a 4x4 are more likely to suffer damage or injury*… Which makes statement ii false.

And finally, consider the third statement: *The majority of four-wheel drive vehicles are only used for short journeys*.

What we were actually told was: *In the UK it is estimated that the majority of four-wheel drive vehicles are only used for short journeys to school or office*… Which makes statement iii false. So the only correct answer is **d**.

Now try the **second example**.

The modern 'welfare state' has allowed a generation of children to grow up without a sense of personal responsibility. The state is blamed for everything from crime and taxation levels to problems in school classrooms. People no longer ask themselves if they should bear some of the responsibility when things go wrong, usually the first question which comes to mind is who do I complain to and how much compensation will I get? Young girls find they are better off if they become pregnant as they immediately go onto a housing list and qualify for child support benefit.

Consider the following statements and select the most appropriate answer:

i *The modern 'welfare state' has allowed a generation of children to grow up without a sense of purpose*.

ii *All young girls immediately go onto a housing list*.

iii *The welfare state has allowed a generation of children to grow up without a sense of personal responsibility*.

- a Statements i and iii are correct.
- b Only statement ii is correct.
- c Only statement iii is correct.
- d All of the statements are incorrect.

Once again you must answer based on what you are told, not what you do or do not agree with.

Exercice	Answer
Numerical Reasoning Example 2 (page 24)	d
Verbal Reasoning Example 2 (page 26)	c

Tips

Read and Practice. Use Maths exercise books, workbooks and booklets. There are many other such books available in bookshops or on the internet.

3.24 PILAPT

To do the PILAPT (Pilot Aptitude Test), which tests coordination, you will need to be familiar with the use of a computer joystick.